

**2024/FYUG/ODD/SEM/
STADSC-102T/145**

FYUG Odd Semester Exam., 2024

**STATISTICS
(1st Semester)**

Course No. : STADSC-102T

(Calculus)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

UNIT—I

1. Answer any *two* from the following : $2 \times 2 = 4$

(a) Define continuity of a function at a point. Show that $f(x) = x^2$ is continuous at $x = a$.

(b) Evaluate :

$$\lim_{x \rightarrow \infty} x - \sqrt{x^2 + x}$$

(c) Let $f(x) = |x|$. Show that $f(x)$ is not differentiable at $x = 0$.

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(Turn Over)

(2)

2. (a) What is homogeneous function? State and prove Euler's theorem. 1+4=5

- (b) (i) Show that $\lim_{x \rightarrow 0} \cos \frac{1}{x}$ does not exist. 3

- (ii) Evaluate : 2

$$\lim_{x \rightarrow 0} \frac{x^2 \sin \frac{1}{x}}{\sin x}$$

OR

3. (a) (i) A function $f(x)$ is defined as

$$f(x) = \begin{cases} x^2, & 0 < x < 1 \\ x, & 1 \leq x < 2 \\ \frac{1}{4}x^3, & 2 \leq x < 3 \end{cases}$$

Check the continuity of $f(x)$ at $x=1$ and $x=2$. 4

- (ii) Define removable discontinuity. 1

- (b) State and prove Leibnitz's theorem. 5

UNIT—II

4. Answer any two from the following : 2×2=4

- (a) Define maxima and minima of a function $f(x)$ at $x=c$.
 (b) Define convexity and concavity of a function.
 (c) Define Jacobian.

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(3)

5. (a) Solve : 5

$$xyp + y^2q = 2xy - 2x^2$$

- (b) If

$$u = \frac{x^2 + y^2 + z^2}{x}, v = \frac{x^2 + y^2 + z^2}{y}, w = \frac{x^2 + y^2 + z^2}{z}$$

find Jacobian of $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. 5

OR

6. (a) (i) Define points of inflection and singular points. 3

- (ii) Show that maximum value of

$$x + \frac{1}{x}$$

is less than its minimum. 2

- (b) If

$$x_1 + x_2 + x_3 = u$$

$$x_2 + x_3 = uv$$

$$x_3 = uvw$$

then find

$$\frac{\partial(x_1, x_2, x_3)}{\partial(u, v, w)}$$

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UNIT—III

7. Answer any two from the following : $2 \times 2 = 4$

(a) Prove that

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

(b) Define gamma and beta functions.

(c) Show that $\Gamma(n+1) = n!$.

8. (a) Show that

$$\sqrt{\pi} = 2^{2m-1} \frac{\Gamma(m) \Gamma\left(m + \frac{1}{2}\right)}{\Gamma(2m)} \quad 5$$

(b) Prove that

$$\beta(m, n) = \frac{\Gamma(m) \Gamma(n)}{\Gamma(m+n)} \quad 5$$

OR

9. (a) Prove that

$$\int_0^1 \frac{x^2}{(1-x^4)} dx \cdot \int_0^1 \frac{1}{(1+x^4)} dx = \frac{\pi}{4\sqrt{2}} \quad 5$$

(b) Evaluate :

$$\iint \sqrt{4x^2 - y^2} dx dy \quad 5$$

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(5)

UNIT—IV

10. Answer any two from the following : $2 \times 2 = 4$

(a) What is exact differential equation?

(b) Define homogeneous and non-homogeneous differential equation.

(c) Solve :

$$(x^2 - 2y)dx + (y^2 - 2x)dy = 0$$

11. (a) Find the differential equation for the following : $2+3=5$

(i) $y = A \sin x + B \cos x$

(ii) $y = e^{-x}(A \cos x + B \sin x)$

(b) Solve the following : $2+3=5$

(i) $x\sqrt{1-y^2}dx + y\sqrt{1-x^2}dy = 0$

(ii) $\frac{dy}{dx} = \frac{6x-2y-7}{2x+3y-6}$

OR

12. (a) Solve : 5

$$(x^3 + y^3)dx - xy^2dy = 0$$

(b) Solve : 5

$$(1+x^2)\frac{dy}{dx} + y = \tan^{-1}x$$

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(6)

UNIT—V

13. Answer any *two* from the following : $2 \times 2 = 4$

- (a) What is linear differential equation?
- (b) Define auxiliary equation of a differential equation.
- (c) State degree and order of partial differential equation.

14. (a) Solve : 5

$$(D^2 + 4)y = x^2$$

(b) Solve : 5

$$\frac{d^2y}{dx^2} + y = \cos 2x$$

OR

15. (a) Solve : 5

$$(D^2 - 2D + 1)y = x^2 e^{3x}$$

- (b) Formulate a partial differential equation by eliminating a and b from

$$z = (x + a)(y + b) \quad 5$$

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